# Year 13 Further Mathematics – Complex Numbers

West Kirby Grammar School

**Learning about this topic is important because:**
Studying complex numbers matters because they unlock a deeper, more powerful way of understanding patterns that real numbers alone can't capture. Complex numbers let us model oscillations, waves, electrical circuits, quantum mechanics, and rotations in a clean, elegant language. The ideas studied in this chapter form a toolkit that shows how algebra and geometry intertwine, and they underpin huge parts of modern science, engineering, and pure mathematics.

**This builds on:** Complex Number Y12FM, Argand diagrams Y12FM, Proof by induction Y12FM, Binomial expansion Y12, Series Y13

**This leads to:** Computer Science, Mathematics or Engineering in Further Education

**We will learn:**

- Express a complex number in exponential form
- Multiply and divide complex numbers in exponential form
- Understand de Moivre's theorem
- Use de Moivre's theorem to derive trigonometric identities
- Use de Moivre's theorem to find sums of series
- Know how to solve completely equations of the form
- $z^n - a - ib = 0$, giving special attention to cases where $a = 1$ and $b = 0$
- Use complex roots of unity to solve geometric problems

**We will develop/practise skills including:**

- Use and apply standard techniques: Select and correctly carry out routine procedures; and accurately recalling facts, terminology and definitions.
- Reason, interpret and communicate mathematically: Construct rigorous mathematical arguments, make deductions and inferences, assess the validity of mathematical arguments, explaining your reasoning; and using mathematical language and notation correctly.
- Solve problems within mathematics and in other contexts: Translate problems in mathematical and non-mathematical contexts into mathematical processes, use mathematical models and interpret solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations and explain how to refine them.

**Some of the vocabulary that we will use includes:**
Exponential, Euler's relation, modulus-argument, De Moivre's Theorem, prove, proof by induction, binomial expansion, series, roots, nth roots of unity.

**You could learn more about this topic by:**
Completing all exercises, mixed exercises, challenges and review questions from the textbook, seeking additional support from your teachers at lunchtime drop-in sessions, searching online for the topic and GeoGebra to find relevant visual and interactive tools for understanding

# Year 13 Further Mathematics – Complex Numbers

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| | new concepts, using free question banks and revision tools on websites such as MathsGenie and Physics Maths Tutor. |
| Your teacher will assess your knowledge & understanding throughout the topic by looking at your work, questioning, discussion and giving you feedback in lots of different ways. The work you produce in class (your file), homework, independent study and regular assessment all form the key pieces of work in this topic. | |

# Year 13 Further Mathematics – Series

West Kirby Grammar School

**Learning about this topic is important because:**
Learning about series matters because they reveal the hidden structure behind complicated mathematical expressions, making problems that look messy suddenly become manageable. They are a powerful way to approximate curves, solve differential equations, and understand how functions behave near zero.

**This builds on:** Sequences and Series Y12, Series Y12FM, Partial Fractions Y13, Differentiation Y12/Y13

**This leads to:** Calculus, differential equations, mathematical modelling, and even physics and engineering in Further Education

**We will learn:**

- Understand and use the method of differences to sum finite series
- Find and use higher derivatives of functions
- Know how to express functions as an infinite series in ascending powers using Maclaurin series expansion
  Be able to find the series expansions of compound functions

**We will develop/practise skills including:**

- Use and apply standard techniques: Select and correctly carry out routine procedures; and accurately recalling facts, terminology and definitions.
- Reason, interpret and communicate mathematically: Construct rigorous mathematical arguments, make deductions and inferences, assess the validity of mathematical arguments, explaining your reasoning; and using mathematical language and notation correctly.
- Solve problems within mathematics and in other contexts: Translate problems in mathematical and non-mathematical contexts into mathematical processes, use mathematical models and interpret solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations and explain how to refine them.

**Some of the vocabulary that we will use includes:**
Method of differences, Sum of, partial fractions, higher derivatives, cancel out, Maclaurin series, infinite sum, approximation, expansion.

**You could learn more about this topic by:**
Completing all exercises, mixed exercises, challenges and review questions from the textbook, seeking additional support from your teachers at lunchtime drop-in sessions, searching online for the topic and GeoGebra to find relevant visual and interactive tools for understanding new concepts, using free question banks and revision tools on websites such as MathsGenie and Physics Maths Tutor.

# Year 13 Further Mathematics – Series

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# Year 13 Further Mathematics – Methods in Calculus

West Kirby Grammar School

**Learning about this topic is important because:**
Methods in calculus enables you to understand how mathematics behaves at its limits. Improper integrals teach you how to handle infinite intervals and functions with singularities, which is essential in probability theory, physics, and any situation involving infinite processes. Inverse trigonometric functions appear naturally when dealing with geometry, arc lengths, substitution techniques, and integrals that don't simplify in ordinary ways, so being able to differentiate and integrate them gives you access to a much wider class of problems.

**This builds on:** Integration Y12/13, Trigonometry Y13, Differentiation Y13, Partial Fractions Y13,

**This leads to:** Multivariable calculus, complex integration, and mathematical modelling in Maths, Physics or Engineering in Further Education.

**We will learn:**

- Evaluate improper integrals
- Understand and evaluate the mean value of a function
- Integrate rational functions using trigonometric substitutions
- Integrate using partial fractions

**We will develop/practise skills including:**

- Use and apply standard techniques: Select and correctly carry out routine procedures; and accurately recalling facts, terminology and definitions.
- Reason, interpret and communicate mathematically: Construct rigorous mathematical arguments, make deductions and inferences, assess the validity of mathematical arguments, explaining your reasoning; and using mathematical language and notation correctly.
- Solve problems within mathematics and in other contexts: Translate problems in mathematical and non-mathematical contexts into mathematical processes, use mathematical models and interpret solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations and explain how to refine them.

**Some of the vocabulary that we will use includes:**
Improper integrals, infinite, undefined, convergent, divergent, Mean value, average value, inverse trigonometric functions, arcsin, arccos, arctan, implicit differentiation, partial fractions.

**You could learn more about this topic by:**
Completing all exercises, mixed exercises, challenges and review questions from the textbook, seeking additional support from your teachers at lunchtime drop-in sessions, searching online for the topic and GeoGebra to find relevant visual and interactive tools for understanding

# Year 13 Further Mathematics – Methods in Calculus

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| | new concepts, using free question banks and revision tools on websites such as MathsGenie and Physics Maths Tutor. |
| Your teacher will assess your knowledge & understanding throughout the topic by looking at your work, questioning, discussion and giving you feedback in lots of different ways. The work you produce in class (your file), homework, independent study and regular assessment all form the key pieces of work in this topic. | |

# Year 13 Further Mathematics – Volumes of Revolution

West Kirby Grammar School

**Learning about this topic is important because:**
Studying volumes of revolution matters because it turns abstract ideas about area, curves, and integration into a powerful method for finding the volume of real 3-D objects, from bowls and bottles to engineering components and physical models. It prepares you for university topics like multivariable calculus, surface integrals, and modelling in physics and engineering, where rotating shapes and continuous volume calculations are everyday tools.

**This builds on:** Volumes of revolution Y12FM, Integration Y12/13, Parametric equations Y13

**This leads to:** Multivariable calculus in Further Education and also in Physics and Engineering in Further Education.

**We will learn:**

- Find volumes of revolution around the x-axis
- Find volumes of revolution around the y-axis
- Find volumes of revolution for curves defined parametrically
- Model real-life applications of volumes of revolution

**We will develop/practise skills including:**

- Use and apply standard techniques: Select and correctly carry out routine procedures; and accurately recalling facts, terminology and definitions.
- Reason, interpret and communicate mathematically: Construct rigorous mathematical arguments, make deductions and inferences, assess the validity of mathematical arguments, explaining your reasoning; and using mathematical language and notation correctly.
- Solve problems within mathematics and in other contexts: Translate problems in mathematical and non-mathematical contexts into mathematical processes, use mathematical models and interpret solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations and explain how to refine them.

**Some of the vocabulary that we will use includes:**
Integration, rotated, limits, about the x-axis, about the y-axis, region, solid generated, parametric equation, cartesian equation, calculus, model.

**You could learn more about this topic by:**
Completing all exercises, mixed exercises, challenges and review questions from the textbook, seeking additional support from your teachers at lunchtime drop-in sessions, searching online for the topic and GeoGebra to find relevant visual and interactive tools for understanding

# Year 13 Further Mathematics – Volumes of Revolution

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| | new concepts, using free question banks and revision tools on websites such as MathsGenie and Physics Maths Tutor. |
| Your teacher will assess your knowledge & understanding throughout the topic by looking at your work, questioning, discussion and giving you feedback in lots of different ways. The work you produce in class (your file), homework, independent study and regular assessment all form the key pieces of work in this topic. | |

# Year 13 Further Mathematics – Polar Coordinates

West Kirby Grammar School

**Learning about this topic is important because:**
Studying polar coordinates is important because they let you describe curves, motion, and regions that are awkward or impossible to express neatly in Cartesian form, especially anything involving rotation, spirals, or circular symmetry. They prepare you for university topics like multivariable calculus, vector fields, electromagnetism, and differential equations, where switching between coordinate systems is a fundamental skill.

**This builds on:** Integration Y13, Argand diagrams Y12FM, Trigonometry Y12/13, differentiation Y12/13

**This leads to:** In further education, polar coordinates expand into multivariable calculus, vector calculus, and coordinate transformations

**We will learn:**

- Understand and use polar coordinates
- Convert between polar and Cartesian coordinates
- Sketch curves with r given as a function of θ
- Find the area enclosed by a polar curve
- Find tangents parallel to, or at right angles to, the initial line

**We will develop/practise skills including:**

- Use and apply standard techniques: Select and correctly carry out routine procedures; and accurately recalling facts, terminology and definitions.
- Reason, interpret and communicate mathematically: Construct rigorous mathematical arguments, make deductions and inferences, assess the validity of mathematical arguments, explaining your reasoning; and using mathematical language and notation correctly.
- Solve problems within mathematics and in other contexts: Translate problems in mathematical and non-mathematical contexts into mathematical processes, use mathematical models and interpret solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations and explain how to refine them.

**Some of the vocabulary that we will use includes:**
Pole, initial line, cartesian form, polar form, trigonometry, quadrant, circle, half line, spiral, egg shape, dimple, tangents, locus, argand diagram, integration, area, region, sector, calculus, cartoid, loop, limits, intersection, parallel, perpendicular.

**You could learn more about this topic by:**
Completing all exercises, mixed exercises, challenges and review questions from the textbook, seeking additional support from your teachers at lunchtime drop-in sessions, searching online for the topic and GeoGebra to find relevant visual and interactive tools for understanding

# Year 13 Further Mathematics – Polar Coordinates

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| | new concepts, using free question banks and revision tools on websites such as MathsGenie and Physics Maths Tutor. |
| Your teacher will assess your knowledge & understanding throughout the topic by looking at your work, questioning, discussion and giving you feedback in lots of different ways. The work you produce in class (your file), homework, independent study and regular assessment all form the key pieces of work in this topic. | |

# Year 13 Further Mathematics – Hyperbolic Functions

West Kirby Grammar School

**Learning about this topic is important because:**
Learning hyperbolic functions is important because they behave like a bridge between algebra, calculus, and real-world modelling, giving you tools that ordinary trig functions can't provide.

**This builds on:** Exponentials Y12, Inverse Trigonometric functions Y13, Differentiation Y13, Integration Y13, Methods in calculus Y13FM

**This leads to:** Maths, Physics and Engineering in Further Education

**We will learn:**

- Understand the definitions of hyperbolic functions
- Sketch the graphs of hyperbolic functions
- Understand and use the inverse hyperbolic functions
- Prove identities and solve equations using hyperbolic functions
  Differentiate and integrate hyperbolic functions

**We will develop/practise skills including:**

- Use and apply standard techniques: Select and correctly carry out routine procedures; and accurately recalling facts, terminology and definitions.
- Reason, interpret and communicate mathematically: Construct rigorous mathematical arguments, make deductions and inferences, assess the validity of mathematical arguments, explaining your reasoning; and using mathematical language and notation correctly.
- Solve problems within mathematics and in other contexts: Translate problems in mathematical and non-mathematical contexts into mathematical processes, use mathematical models and interpret solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations and explain how to refine them.

**Some of the vocabulary that we will use includes:**
Sinh, cosh, tanh, exponential, natural logarithms, cosech, sech, coth, odd function, even function, inverse trigonometric functions, arsinh, arcosh, artanh, trigonometric Identity, Osborn's rule, differentiate, percentage error, integration.

**You could learn more about this topic by:**
Completing all exercises, mixed exercises, challenges and review questions from the textbook, seeking additional support from your teachers at lunchtime drop-in sessions, searching online for the topic and GeoGebra to find relevant visual and interactive tools for understanding new concepts, using free question banks and revision tools on websites such as MathsGenie and Physics Maths Tutor.

# Year 13 Further Mathematics – Hyperbolic Functions

Your teacher will assess your knowledge & understanding throughout the topic by looking at your work, questioning, discussion and giving you feedback in lots of different ways. The work you produce in class (your file), homework, independent study and regular assessment all form the key pieces of work in this topic.

# Year 13 Further Mathematics – Methods in Differential Equations

West Kirby Grammar School

**Learning about this topic is important because:**
Studying first-order and second-order differential equations is important because they're the mathematical language of change, letting you model how systems evolve in time — from population growth to electrical circuits to mechanical vibrations. Knowing the nature of the roots of the characteristic equation matters because it tells you exactly how a system behaves: whether it oscillates, decays smoothly, or grows without bound.

**This builds on:** Differential Equations Y13, Differentiation Y13, Integration Y13, Complex numbers Y12FM/13FM

**This leads to:** Physics, Engineering, Economics or Mathematics in Further Education

**We will learn:**

- Solve first-order differential equations using an integrating factor
- Solve second-order homogeneous differential equations using the auxiliary equation
- Solve second-order non-homogeneous differential equations using the complimentary function and the particular integral
- Find particular solutions to differential equations using given boundary conditions

**We will develop/practise skills including:**

- Use and apply standard techniques: Select and correctly carry out routine procedures; and accurately recalling facts, terminology and definitions.
- Reason, interpret and communicate mathematically: Construct rigorous mathematical arguments, make deductions and inferences, assess the validity of mathematical arguments, explaining your reasoning; and using mathematical language and notation correctly.
- Solve problems within mathematics and in other contexts: Translate problems in mathematical and non-mathematical contexts into mathematical processes, use mathematical models and interpret solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations and explain how to refine them.

**Some of the vocabulary that we will use includes:**
First-order, second-order, separating the variables, general solution, particular solution, product rule, integrating factor, boundary condition, arbitrary constant, homogeneous, nature of root, auxiliary equation, distinct real roots, repeated root, complex

**You could learn more about this topic by:**
Completing all exercises, mixed exercises, challenges and review questions from the textbook, seeking additional support from your teachers at lunchtime drop-in sessions, searching online for the topic and GeoGebra to find relevant visual and interactive tools for understanding

# Year 13 Further Mathematics – Methods in Differential Equations

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| conjugate root, complementary function, particular integral, boundary conditions. | new concepts, using free question banks and revision tools on websites such as MathsGenie and Physics Maths Tutor. |
| Your teacher will assess your knowledge & understanding throughout the topic by looking at your work, questioning, discussion and giving you feedback in lots of different ways. The work you produce in class (your file), homework, independent study and regular assessment all form the key pieces of work in this topic. | |

# Year 13 Further Mathematics – Modelling with Differential Equations

West Kirby Grammar School

**Learning about this topic is important because:**
Learning about modelling with differential equations is important because these ideas show you how mathematics describes real physical systems, turning abstract equations into tools for understanding motion, energy, and stability. Together, these topics form the foundation of how engineers design structures, how physicists analyse oscillations, and how mathematicians model dynamic systems across science and technology.

**This builds on:** Differential equations Y13/Y13FM, differentiation Y13, Integration Y13, Kinematics Y12

**This leads to:** Physics, Engineering or Mathematics in Further Education

**We will learn:**

- Model real-life situations with first-order differential equations
- Use differential equations to model simple harmonic motion
- Model damped and forced oscillations using differential equations
- Model real-life situations using coupled first-order differential equations

**We will develop/practise skills including:**

- Use and apply standard techniques: Select and correctly carry out routine procedures; and accurately recalling facts, terminology and definitions.
- Reason, interpret and communicate mathematically: Construct rigorous mathematical arguments, make deductions and inferences, assess the validity of mathematical arguments, explaining your reasoning; and using mathematical language and notation correctly.
- Solve problems within mathematics and in other contexts: Translate problems in mathematical and non-mathematical contexts into mathematical processes, use mathematical models and interpret solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations and explain how to refine them.

**Some of the vocabulary that we will use includes:**
First order, kinematics, displacement, velocity, acceleration, simple harmonic motion, proportional, centre of oscillation, angular velocity, dot notation, amplitude, period, equilibrium position, stationary point, damped, auxiliary equation, distinct, equal or complex roots, critical damping, light damping, heavy damping,

**You could learn more about this topic by:**
Completing all exercises, mixed exercises, challenges and review questions from the textbook, seeking additional support from your teachers at lunchtime drop-in sessions, searching online for the topic and GeoGebra to find relevant visual and interactive tools for understanding

# Year 13 Further Mathematics – Modelling with Differential Equations

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| general solution, particular solution, forced harmonic motion, coupled, connected, simultaneous equations, elimination, dependent variable. | new concepts, using free question banks and revision tools on websites such as MathsGenie and Physics Maths Tutor. |
| Your teacher will assess your knowledge & understanding throughout the topic by looking at your work, questioning, discussion and giving you feedback in lots of different ways. The work you produce in class (your file), homework, independent study and regular assessment all form the key pieces of work in this topic. | |